# Photoinduced adiabatic state transfer in low-dimensional electronic systems

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To perform an adiabatic state transfer which brings about a change in the electron Berry phase effect, a manipulable slow degree of freedom should be introduced in an electron system. We show that the use of a laser field provides a solution to this problem; we derive effective electronic Hamiltonians for one- and two-dimensional systems in a driving laser field whose amplitude slightly depends on time. We show that in the resulting electron-only theory, the renormalized hopping energy turns into an adiabatically tunable variable, which in turn will directly affect the Berry phase properties of the electron system. As an important application of this effect, we discuss how our scheme will open up an efficient way to optically control the electric polarization of the system.

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## I. INTRODUCTION

Over the years, the condensed matter physics community has come to grasp the immense potential which the Berry phase effect (BPE) of Bloch electrons embodies.[1–3] By the term BPE, one has in mind various response functions which are solely determined by the momentum space Berry phase structure of the system's Hamiltonian. Some typical examples are the electric polarization,[4–6] the intrinsic spin Hall conductivity,[7] and the anomalous Hall conductivity.[8] The origins of BPEs can be traced to the topology of the momentum space, or in a broader sense, to that of the manifold on which the system is defined. Information on the latter enters into electron dynamics primarily through the Berry connection $\mathcal{A}_n(\mathbf{k})=i\langle v_{n\mathbf{k}}|\nabla_k|v_{n\mathbf{k}}\rangle$, which plays the role of a generalized vector potential (here $v_{n\mathbf{k}}$ represents the cell-periodic part of the Bloch wave functions in the $n$th band). The curl of this quantity is the curvature of the aforementioned manifold, while its contour integral yields momentum space Berry phases. The relevance of such nontrivial phases to electron dynamics was noted early on in the work of Karplus and Luttinger, who identified these objects with the anomalous velocity of the Bloch electrons.[9] The full connection to topology was understood much later within a refined semiclassical framework of electron wave-packet dynamics.[10] Generally an adiabatic parameter is associated with the Berry phase. A well-known example is found in a system of a spin 1/2 object in slightly time-dependent magnetic field, where the direction of the magnetic field is the adiabatic parameter.[11] In the Bloch electron case, the wave number $\mathbf{k}$ plays the role of the adiabatic parameter.

Electron systems which exhibit nontrivial BPEs are often referred to as topological materials. Devising schemes to induce adiabatic state transfers in these class of materials have been a subject of considerable interest, since that would enable one to directly *manipulate* the BPEs of the system, which is otherwise a fixed material property. Since the wave number is intrinsic and not externally manipulable, the adiabatic state transfer in electron systems demands the addition of a tunable extra dimension $\lambda$ to the momentum space. Thus the space for the adiabatic parameters is enlarged; it is now spanned by the wave number and the slow variable $\lambda$. Then the adiabatic state transfer is realized by varying $\lambda$, and induces a Berry curvature [to be defined later in Eq. (13)] which encodes the nontrivial topology of the parameter space. The concept of adiabatic state transfer lies at the heart of the principle of charge pumping in one-dimensional electron systems,[12] as well as the Berry phase theory of electric polarization.[4–6]

The generic theoretical method to deal with the adiabatic state transfer is to rely on a parametric Hamiltonian $H(\lambda)$ with the slow variable $\lambda$. A central question here is how to induce into a given electron system a suitable adiabatic parameter *that can be tuned externally in a feasible way*. In this regard it is worth noting that adiabatic parameters which appear in the BPE literature often fall short of this prerequisite condition. In the Berry phase theory of polarization, for instance, they are usually treated as auxiliary variable,[4–6] whose adiabatic changes are purely conceptual. In other words, they are introduced solely as a mathematical device for evaluating the electric polarization. A typical example is seen in Ref. 13 where the staggered on-site potential, a material constant, is chosen to play this role.

The purpose of the present article is to lay the basis for a series of work aiming to establish a feasible scheme toward the adiabatic control of the BPEs in topological materials. Our basic strategy is to realize the adiabatic state transfer through the use of a driving laser field applied to the electron systems. We show that under the appropriate conditions, the effect of the coupling to the laser field whose amplitude depends on time provides us with the adiabatic parameters suitable for the purpose; these appear in a form of a renormalized electron hopping energy that depends on the laser amplitude. Thus, for those systems in which the hopping energies govern the Berry phases, an adiabatic variation of the laser amplitude (i.e., slow on the scale of the inverse laser frequency) offers us a straightforward method to tune the BPEs. We show that when combined with the Berry phase theory of polarization, this general scheme can be used to gain an optical control of the electric polarization. Another direction which can be pursued is to employ the strategy to change the topological order[14] of the system in question, e.g., to drive a nontopological insulator into a topological insulator. This application, which we believe to be of considerable

 

interest in view of the current focus on topological insulators, will be detailed in a separate publication.[15]

## II. INTRODUCTION OF SLOW VARIABLE FOR ADIABATIC STATE TRANSFER AND DERIVATION OF EFFECTIVE HAMILTONIAN

To identify the adiabatic parameter which meets our purpose, we will take advantage of the fact that a driving laser field with frequency $\omega$ and *fixed* amplitude $E_0$ effectively turns the hopping energy of the Bloch electron $t_0$ into $t_0 J_0(\lambda_0)$. The *constant* $\lambda_0$ is defined as $\lambda_0 \equiv eaE_0/\hbar\omega$, where $a$ and $e$ are the lattice constant and the unit charge, respectively, and $J_0(\lambda_0)$ is the Bessel function of order zero.[16] This laser-induced modification of the hopping energy already assumes a central role in the study of dynamic localization[17] and the miniband collapsing in superlattices.[18] However, previous methods of deriving the aforementioned $t_0 J_0(\lambda_0)$ were specific to one-dimensional systems. Since much of the recent progress on BPEs involve two-dimensional systems, we need to devise a systematic method, applicable regardless of the dimensionality that correctly incorporates the effect of the laser field into the low energy description of electron systems. This we do by adopting the Floquet theory of nonequilibrium dynamics.[19]

Let us begin with a generic Hamiltonian $H=H^{el}+H^{int}(t)$, where $H^{el}$ and $H^{int}(t)$ each describe Bloch electrons in a low-dimensional electron system with a finite band gap and an interaction of the electrons with the driving laser field whose amplitude, $E_T$, depends on time. We assume that the adiabatic approximation is valid, i.e., that the temporal variation of the laser amplitude is much slower than the inverse frequency $\omega^{-1}$ of the laser field. The problem then acquires a temporal periodicity, which allows to make use of the Floquet theory;[20,21] the latter converts the time-dependent problem into a time-independent eigenvalue problem for the Floquet Hamiltonian $\tilde{H}$. The eigenvalues of this infinite-dimensional Hamiltonian are referred to as quasienergies. The Floquet Hamiltonian $\tilde{H}$ consists of block Hamiltonians $\tilde{H}^{(mn)}$ labeled by integers $m$ and $n$. The matrix element for each block reads

$$[\tilde{H}^{(mn)}]_{\alpha\beta} = \frac{1}{2\pi}\int_0^{2\pi} e^{i(m-n)\theta}[H^{el}]_{\alpha\beta} d\theta + n\hbar\omega\delta_{\alpha\beta}\delta_{nm}, \quad (1)$$

where $\{|\alpha\rangle\}$ represents a suitable set of basis for the electronic wave function. The Hilbert space of $\tilde{H}$ is spanned by both the electrons *and* photons of the energy $\hbar\omega$. The diagonal block $H^{(nn)}$ represents an $n$-photon sector. When the mixture between the $n$- and $n\pm 1$-photon sectors is negligible (which is the case when the below gap excitation energy $\hbar\omega$ exceeds the electron bandwidth),[22] all information on the composite electron-photon system is encoded solely within the block $\tilde{H}^{(00)}$. This follows because in this case the quasienergy spectra of $\tilde{H}$ read $\tilde{\varepsilon}_k^{(00)}+n\hbar\omega$, where $\tilde{\varepsilon}_k^{(00)}$ is the energy bands given by $\tilde{H}^{(00)}$. Under these conditions, the hopping energy appearing in $\tilde{H}^{(00)}$, i.e., $\tilde{t}_0(\lambda) \equiv t_0 J_0(\lambda)$, should be regarded as the expression for the effective hopping integral. We then apply the adiabatic strategy; we slowly vary the laser amplitude $E_T$ (or $\lambda \equiv eaE_T/\hbar\omega$) and regard $\tilde{t}_0(\lambda)$ as the slow variable.

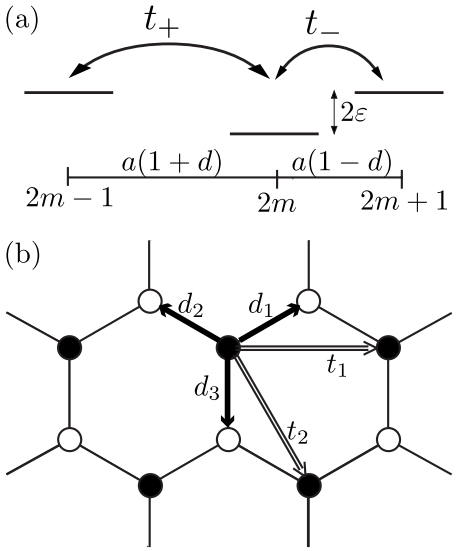


FIG. 1. (a) One- and (b) two-dimensional lattices considered in this study.

Having explained the general outline of our program for deriving the effective Hamiltonian, we now apply it to a one-dimensional electron system. Consider, then, a tight-binding electron system that has both staggered on-site energy ($\pm\varepsilon$) and bond length alternation $a_\pm = a(1\pm d)$, where $0<d<1$ [see Fig. 1(a)]. Hopping energies between neighboring sites, $t_\pm$, also alternate in accord with the alternation of $a_+$ and $a_-$. We will later see that this choice of Hamiltonian gives rise to nontrivial BPEs. We couple the electrons to a driving laser field with time-dependent amplitude and a polarization which aligns with linear extent of the one-dimensional lattice. The total Hamiltonian reads

$$H_{1d} = H_{1d}^{el} + H_{1d}^{int}, \quad (2)$$

$$H_{1d}^{el} = \frac{t_-}{2}\sum_m c_{2m}^\dagger c_{2m+1} + \frac{t_+}{2}\sum_m c_{2m-1}^\dagger c_{2m} + \text{H.c.} + \varepsilon\sum_n (-1)^n c_n^\dagger c_n, \quad (3)$$

$$H_{1d}^{int} = eE(t)\sum_n x_n c_n^\dagger c_n, \quad (4)$$

where $c_i$ and $c_i^\dagger$ are electron annihilation and creation operators at site $i$. As mentioned before, we will assume that the adiabatic approximation is applicable. After a gauge transformation which eliminates $H_{1d}^{int}$, the Floquet theorem provides us with

$$\tilde{H}_{1d}^{(00)}(\lambda) = \sum_k (c_k^\dagger \quad c_{k+\pi}^\dagger)\tilde{\mathcal{H}}_{1d}(\lambda)\begin{pmatrix} c_k \\ c_{k+\pi} \end{pmatrix}, \quad (5)$$

$$\tilde{\mathcal{H}}_{1d}(\lambda) = \varepsilon\sigma_x + \frac{\tilde{t}_-(\lambda) - \tilde{t}_+(\lambda)}{2}\sin k\sigma_y + \frac{\tilde{t}_-(\lambda) + \tilde{t}_+(\lambda)}{2}\cos k\sigma_z, \tag{6}$$

where $\sigma_i (i=x,y,z)$ are the Pauli matrices. The operator $c_k(c_k^\dagger)$ is the Fourier transform of $c_i(c_i^\dagger)$. The coefficients $\tilde{t}_\pm(\lambda)$ are the effective hopping energies, defined by

$$\tilde{t}_\pm(\lambda) = \frac{t_+ + t_-}{2(1 \pm d)^2} J_0[\lambda(1 \pm d)]. \tag{7}$$

Hence we find that the net effect of the laser field has been to simply modify the original hopping energies into $\tilde{t}_\pm(\lambda)$, which can be tuned by adiabatically sweeping the variable $\lambda$, or equivalently, the laser amplitude. We are thus led to identify these modified hopping energies to be the slowly varying degrees of freedom of our problem. In Sec. III, we will examine how an adiabatic manipulation of $\tilde{t}_\pm(\lambda)$ will generate a nontrivial Berry curvature in the extended parameter space $\mathbf{k}$-$\lambda$, which in turn will manifest itself as a sizable change in the electric polarization.

We now repeat the same procedure for a two-dimensional system. To this end we choose an electron system known to exhibit nontrivial BPEs.[13] This model, as depicted in Fig. 1(b), is defined on a honeycomb lattice, which can be viewed as superposition of A and B triangular sublattices [the open and solid dots in Fig. 1(b)]. Once again coupling the electrons to a driving laser field, the total Hamiltonian of the system becomes

$$H_{2d} = H_{2d}^{el} + H_{2d}^{int}, \tag{8}$$

$$H_{2d}^{el} = \sum_{\vec{r}\in A}\sum_i t_0 a^\dagger(\vec{r}) b(\vec{r} + \vec{d}_i) + \text{H.c.} + \Delta\sum_{\vec{r}\in A} a^\dagger(\vec{r})a(\vec{r}) - \Delta\sum_{\vec{r}\in B} b^\dagger(\vec{r})b(\vec{r}), \tag{9}$$

$$H_{2d}^{int} = e\vec{E}(t)\cdot\left[\sum_{\vec{r}\in A}\vec{r}a^\dagger(\vec{r})a(\vec{r}) + \sum_{\vec{r}\in B}\vec{r}b^\dagger(\vec{r})b(\vec{r})\right]. \tag{10}$$

The operators $a(\vec{r})[a^\dagger(\vec{r})]$ and $b(\vec{r})[b^\dagger(\vec{r})]$ each annihilate [create] electrons at site belonging to the A and B sublattices, $t_0$ is the hopping energy between the neighboring sites connecting the A and B sublattices and $2\Delta$ is an on-site energy difference between the A- and B-site electrons.

When the driving laser field has circular polarization $\vec{E}(t) = E_T(\cos\omega t, \pm\sin\omega t)$,[23,24] the Floquet theory generates an effective Hamiltonian $\tilde{H}_{2d}^{(00)}(\lambda)$, in which the hopping energy $t_0$ is modified to $\tilde{t}_0(\lambda) \equiv t_0 J_0(\lambda)$, but is otherwise identical in form to $H_{2d}^{el}$. Therefore, as in the case of $H_{2d}^{el}$, the electronic properties can be described by two-dimensional Dirac-fermionlike Hamiltonians within the Brillouin zone, centered at the wave vectors $K_{\alpha=\pm1} = (\alpha 4\pi/3\sqrt{3}a, 0)$, which are the points where band crossing occurs when $\Delta = 0$.[25] Indeed the expansion of $\tilde{H}_{2d}^{(00)}(\lambda)$ around the $K_\pm$ points yields

$$\tilde{H}_{2d}^{(00)}(\lambda) = \sum_k [a^\dagger(k) \;\; b^\dagger(k)]\tilde{\mathcal{H}}_{2d}(\lambda)\begin{bmatrix} a(k) \\ b(k) \end{bmatrix}, \tag{11}$$

$$\tilde{\mathcal{H}}_{2d}(\lambda) = \gamma(\lambda)(\alpha k_x\sigma_x + k_y\sigma_y) + \Delta\sigma_z, \tag{12}$$

where $a(k)$ and $b(k)$ are the Fourier transform of $a(r)$ and $b(r)$, respectively, and the wave number is measured relative to the $K_\pm$ points. The matrix $\tilde{\mathcal{H}}_{2d}(\lambda)$ is insensitive to the directions of the circular polarization. The Eqs. (11) and (12) suggest that the Fermi velocity $\gamma(\lambda) = (3a/2)\tilde{t}_0(\lambda)$ can be regarded as the slow variable of the system, whose value is slaved to an adiabatic change in the laser amplitude. The fact that the driving field has the circular polarization is essential. We note in passing that applying a driving laser field with a linear polarization no longer gives rise to the Dirac Hamiltonian around the $K_\pm$ points.[26] It then follows that the adiabatic state transfer and the accompanying change in the BPEs cannot be realized. This is because the linear polarization couples anisotropically to the electron on the two-dimensional lattice and breaks the isotropic nature of the Dirac-type spectrum.

The effective Hamiltonians $\tilde{\mathcal{H}}_{1d}(\lambda)$ and $\tilde{\mathcal{H}}_{2d}(\lambda)$ are the central results of this work. They establish how the variation in an externally tunable quantity, the laser amplitude, can be conveniently employed to induce an adiabatic state transfer in the electron system. Here the significance lies in the fact that the latter leads to a nontrivial change in the BPEs exhibited by the system, as we will see in Sec. III. Here it is worth highlighting the difference between the Hamiltonian $\tilde{\mathcal{H}}_{2d}(\lambda)$ and the situation considered in Ref. 13, where the Fermi velocity is fixed while $\Delta$ is a slow variable. The variation in $\Delta$ in Ref. 13 represents hypothetical adiabatic transformation between graphene and boron-nitride sheet. The variable is not intended to be an actual physical process.

We expect that the photoinduced adiabatic state transfer is likely to be observed in systems with a large lattice constant and a small hopping energy. Furthermore, changes in BPEs should become appreciable when $\lambda \sim \mathcal{O}(1)$. Let us then make a simple order estimate, taking all these factors into account. Let the lattice constant and hopping energy be of the order of $a \sim 1$ [Å] and $|t_0| \sim 1$ [eV], respectively, which are the typical values in insulating materials. In order to guarantee the validity of the expression $t_0 J_0(\lambda)$ for the hopping energy, the below gap excitation energy $\hbar\omega$ must be bounded from below by the hopping energy, i.e., $\Delta > \hbar\omega \gtrsim 1$ [eV]. The magnitude of the driving laser field then amounts to the order of $\sim 10^7$ [V/cm]. Materials with smaller hopping energies $t_0 \sim 0.1$ [eV] would thus be preferable. One-dimensional organic compounds as TTF-CA and $(TMTTF)_2PF_6$, described by the one-dimensional Hamiltonian Eq. (3),[27] are possible candidates. A system with larger lattice constant, which may be realized, e.g., in artificially fabricated systems is more promising. For a one-dimensional superlattice with lattice constant $\gtrsim 100-1000$ [Å] and resulting hopping energy $\lesssim 0.1$ [eV], the frequency of the driving field is estimated to fall in between the THz and far-infrared ranges. The field

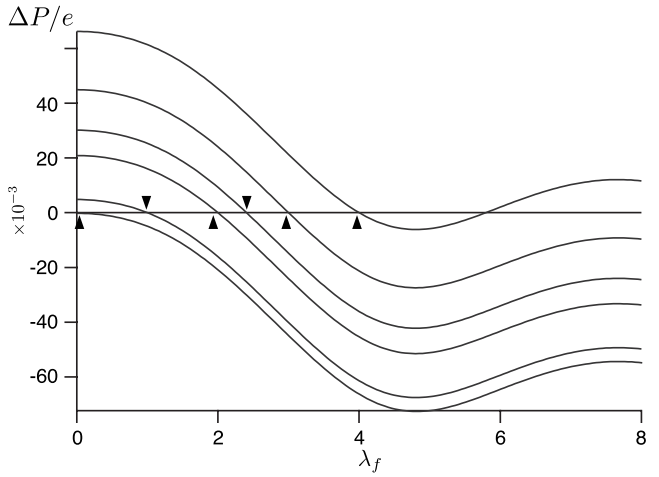


FIG. 2. The polarization $\Delta P/e$ (dipole moment per unit length) in the one-dimensional lattice for $\lambda_i=0.1$, 1.0, 2.0, 2.40 [the first zero point of $J_0(\lambda)$], 3.0 and 4.0, denoted by the solid triangles on the curves. The parameters used in the plots are $\hbar\omega/|t_0|=1.5$, $\varepsilon/|t_0|=2.0$, and $d/a=0.5$.

amplitude is then required to be of the order of $E\sim 10^4$ [V/cm].[29]

## III. MANIFESTATION OF THE BERRY PHASE EFFECT: ELECTRIC POLARIZATION

To demonstrate effectiveness of our theoretical formulation, we calculate the electric polarization on the basis of the Hamiltonians $\tilde{\mathcal{H}}_{1d}(\lambda)$ and $\tilde{\mathcal{H}}_{2d}(\lambda)$. The polarization, as we will see shortly, is a direct manifestation of BPEs in the $\mathbf{k}$-$\lambda$ space. For the purpose, we adopt the Berry phase theory of electric polarization in band insulators. The electric polarization is described in term of the Berry curvature, touched upon in Sec. I, in the enlarged parameter space; this quantity plays the role of a fictitious magnetic field in the parameter space. The $i$th component of the Berry curvature is defined as[4]

$$\mathcal{F}_{n,i}(\mathbf{k},\lambda)=\mathrm{Im}\left\langle \frac{\partial v_{n\mathbf{k}}(\lambda)}{\partial\lambda}\middle|\frac{\partial v_{n\mathbf{k}}(\lambda)}{\partial k_i}\right\rangle. \tag{13}$$

According to the above-mentioned theory, the photoinduced adiabatic state transfer accompanying the variation of $\lambda$ from $\lambda_i$ to $\lambda_f$ gives rise to a change in the electric polarization. In a $D$-dimensional electron system, the $i$th component of the polarization change [$-e(<0)$ is the electron charge] is represented as an integral of the Berry curvature over the Brillouin zone[4–6]

$$\Delta P_i=\frac{2e}{(2\pi)^D}\sum_n\int_{\lambda_i}^{\lambda_f}d\lambda\int_{\mathrm{B.Z.}}d\mathbf{k}\mathcal{F}_{n,i}(\mathbf{k},\lambda), \tag{14}$$

where the summation with respect to $n$ is restricted to occupied bands. This polarization could be observed as the induced current as discussed in Ref. 6. Solving the eigenvalue problems for $\tilde{\mathcal{H}}_{1d}$ and $\tilde{\mathcal{H}}_{2d}$, and, for each case, plugging into Eq. (14) the solution with the lower-energy eigenvalue, we

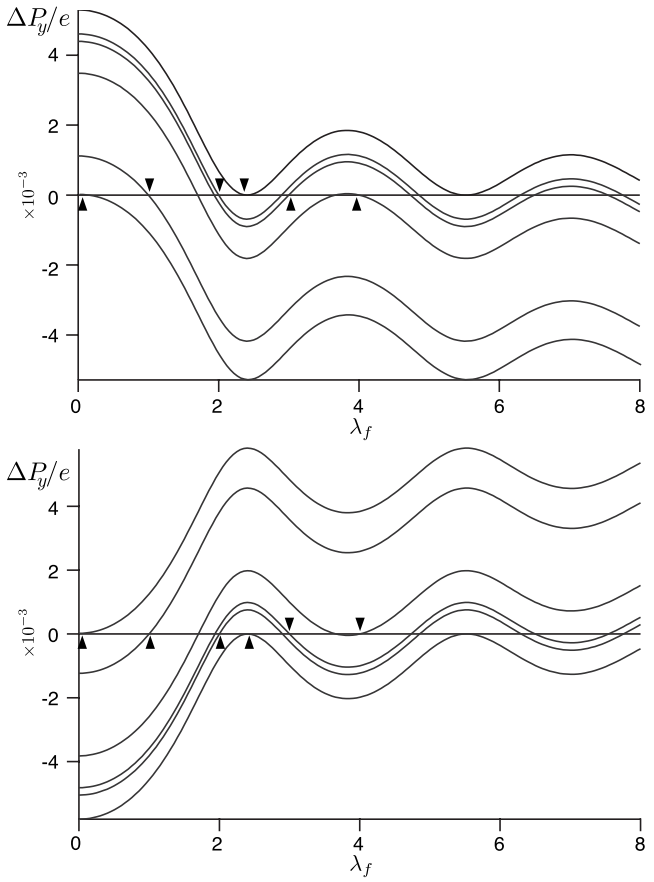


FIG. 3. The polarization $\Delta P/e$ (dipole moment per unit length) in the honeycomb tube with (top) $\nu=+1$ and (bottom) $\nu=-1$ for $\lambda_i=0.1$, 1.0, 2.0, 2.40 [the first zero point of $J_0(\lambda)$], 3.0 and 4.0, denoted by the solid triangles marked on the curves. The parameters used in the plots are $\hbar\omega/|t_0|=1.5$ and $\Delta/|t_0|=2.0$.

arrive at the expression for the electric polarization. For the one-dimensional case, we find

$$\Delta P=\frac{e\varepsilon}{4\pi}\int_{\lambda_i}^{\lambda_f}d\lambda\int dk\frac{1}{4E_{1d}^3}\times[(\tilde{t}_-'+\tilde{t}_+')(\tilde{t}_--\tilde{t}_+)\cos^2 k+(\tilde{t}_-+\tilde{t}_+)(\tilde{t}_-'-\tilde{t}_+')\sin^2 k], \tag{15}$$

with

$$E_{1d}=\sqrt{\varepsilon^2+\left(\frac{\tilde{t}_--\tilde{t}_+}{2}\right)^2\sin^2 k+\left(\frac{\tilde{t}_-+\tilde{t}_+}{2}\right)^2\cos^2 k}. \tag{16}$$

The corresponding expression for the two-dimensional case is

$$\Delta P_j=\sum_{\alpha=\pm}\frac{\alpha e\Delta}{(2\pi)^2}\int d^2k\int_{\lambda_i}^{\lambda_f}d\lambda\frac{\gamma(\lambda)\gamma'(\lambda)}{2E_{2d}^3}\epsilon_{jj'}k_{j'}, \tag{17}$$

with

$$E_{2d}=\sqrt{\gamma^2(\lambda)(k_x^2+k_y^2)+\Delta^2}, \tag{18}$$

where $\{j,j'\}=\{x,y\}$. The prime denotes differentiation with respect to $\lambda$ and $\epsilon_{jj'}$ is the antisymmetric tensor.

Figure 2 shows $\Delta P/e$ in the one-dimensional system as a function of $\lambda_f$ for six initial values of $\lambda_i$, marked with the solid triangles on the curves (here and hereafter, $a=1.0$ [Å] and $t_0=-1.0$ [eV] are used). One can see that the magnitude as well as the sign of the polarization are rather sensitive to the value of $\lambda_i$; observe, e.g., the sign reversal encountered upon increasing $\lambda_f$ for the case of $\lambda_i=4.0$. The nonmonotonic behavior of the polarization originates from the Bessel function in $\tilde{t}_{\pm}(\lambda)$. In the two-dimensional case, we obtain both $\Delta P_j$ along $x(\|\vec{t}_1)$ and $y(\perp\vec{t}_1)$ axes. Although the individual Berry curvature at the $K_{\pm}$ points are nontrivial, the two contributions cancel out. Wrapping the two-dimensional lattice into a tube provides a finite $\Delta P_j$,[13] because the boundary condition forces the wave number to take on discretized values which depend on the chiral index $\nu$. The wrapping is characterized by identifying the origin with the lattice point $\vec{T}=N_1\vec{t}_1+N_2\vec{t}_2$ ($N_1$, $N_2$: integer). The chiral index, defined through the relation $N_1+N_2=3N+\nu$ for an integer $N$, takes the value $\pm 1$ and 0.[28] Here, we set $N_2=0$ and the resulting zigzag tube has a finite $\Delta P_y$. Figure 3 shows $\Delta P_y/e$ for the cases of $\nu=\pm 1$. Note that the tubes with $\nu=0$ have no polarization. Each curve displayed in Fig. 3 corresponds to different values of $\lambda_i$, which are marked with the solid triangles. The overall behavior is similar with that of the one-dimensional case. An interesting observation is the definite sign of the polarization for the entire range of $\lambda_f$ when $\lambda_i$ is the first zero point of $J_0(\lambda)$. One can in fact show that this feature arises whenever $\lambda_i$ takes any zero point of $J_0(\lambda)$.

The electric polarization in the present study should be compared with the conventional polarization within the linear response theory. In the latter case, the electric susceptibility $\chi\sim 1/(\omega^2-\omega_g^2)$ is constant for a fixed excitation frequency $\omega$ and does not depend on the field amplitude. In marked contrast to it, the electric polarization shown in Figs. 2 and 3 exhibits the oscillatory behavior as monotonic increase in $E_T$ for a fixed $\omega$. In particular in the tube case, the sign depends on chirality. These nonlinear features of the electric polarization would not straightforwardly be unveiled when perturbation theories are used.

## IV. CONCLUSION

We have shown that a driving laser field can introduce a tunable adiabatic parameter into the low-energy description of electron systems. The electron hopping energy in the Hamiltonian obtained is the slow variable, which is manipulated by the driving laser amplitude. The photoinduced adiabatic state transfer described by the slow variable alters the electron Berry phase effects. This would open the possibility of advanced control of topological materials. As an illustration of the general structure, electric polarizations in one- and two-dimensional systems are calculated. The electric polarization shown in this study is a fruitful result of the interplay of the Floquet method and the Berry phase theory of electric polarization.

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[29] Generation of the high-intense field, as well as pulse-shaping techniques, in these frequency ranges are current extensive research topics.